

Perhaps We Have Misunderstood the Maxwell's Theory and the Galilean Transformations

Gaobiao xiao, Shanghai Jiao Tong University, China.

gaobiaoxiao@sjtu.edu.cn

Abstract The Einstein's theory of special relativity is based on his two postulates. The first is that the laws of physics are the same in all inertial reference frames. The second is that the velocity of light in the vacuum is the same in all inertial frames. The theory of special relativity is considered to be supported by a large number of experiments. This paper revisits the two postulates according to the new interpretations to the exact solutions of moving sources in the laboratory frame. The exact solutions are obtained using the classic Maxwell's theory, which clearly show that the propagation velocity of the electromagnetic waves of moving sources in the vacuum is not isotropic; the propagation velocity of the electromagnetic waves and the moving velocity of the sources cannot be added like vectors; the transverse Doppler effect is intrinsically included in the fields of the moving sources. The electromagnetic sources are subject to the Newtonian mechanics, while the electromagnetic fields are subject to the Maxwell's theory. We argue that since their behaviors are quite different, it is not a best choice to try to bind them together and force them to undergo the same coordinate transformations as a whole, like that in the Lorentz transformations. Furthermore, the Maxwell's theory does not impose any limitations on the velocity of the electromagnetic waves. To assume that all objects cannot move faster than the light in the vacuum need more examinations. We have carefully checked the main experiment results that were considered as supporting the special relativity. Unfortunately, we found that the experimental results may have been misinterpreted. We here propose a Galilean-Newtonian-Maxwellian relativity, which can give the same or even better explanations to those experimental results.

I. Introduction

In the coordinate system $O(\mathbf{r}, t)$, the Maxwell's equations in the vacuum can be expressed by [1,2,3]

$$\begin{cases} \nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial}{\partial t} \mathbf{B}(\mathbf{r}, t) \\ \nabla \times \mathbf{H}(\mathbf{r}, t) = \frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t} + \mathbf{J}(\mathbf{r}, t) \\ \nabla \cdot \mathbf{B}(\mathbf{r}, t) = 0 \\ \nabla \cdot \mathbf{D}(\mathbf{r}, t) = \rho(\mathbf{r}, t) \end{cases} \quad (1)$$

where $\mathbf{E}(\mathbf{r}, t)$ is the electric field intensity, $\mathbf{H}(\mathbf{r}, t)$ the magnetic field intensity, $\mathbf{D}(\mathbf{r}, t) = \epsilon_0 \mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t) = \mu_0 \mathbf{H}(\mathbf{r}, t)$ are respectively the corresponding flux densities. The fields are generated by the charge density $\rho(\mathbf{r}, t)$ and the related current density $\mathbf{J}(\mathbf{r}, t)$. In the vacuum, the current density is caused by the motion of the charge density, i.e.,

$$\mathbf{J}(\mathbf{r}, t) = \rho(\mathbf{r}, t) \mathbf{v}_\rho(\mathbf{r}, t) \quad (2)$$

where $\mathbf{v}_\rho(\mathbf{r}, t)$ is the velocity of $\rho(\mathbf{r}, t)$. In the vacuum, the permittivity ϵ_0 and the permeability μ_0 are constants.

The scalar potential $\phi(\mathbf{r}, t)$ satisfies the wave equation

$$\nabla^2 \phi(\mathbf{r}, t) - \frac{1}{c_0^2} \frac{\partial^2 \phi(\mathbf{r}, t)}{\partial t^2} = f(\mathbf{r}, t) \quad (3)$$

where $f(\mathbf{r}, t)$ is the excitation source, $c_0 = 1/\sqrt{\mu_0 \epsilon_0}$ is also a constant. It is conventionally considered that c_0 is the wave velocity since (3) is in the same form of the wave equation in the classic mechanics. However, it is very important to note that the Maxwell's theory does not state that the velocity of the

electromagnetic waves in the vacuum is always c_0 . For a single pulse charge existing at the point \mathbf{r}_1 and the time instant t_1 , its fields at the point \mathbf{r} and time t is determined by the Green's function [4, 5, 6]

$$g(\mathbf{r}, \mathbf{r}_1, t) = \frac{1}{4\pi|\mathbf{r} - \mathbf{r}_1|} \delta(t - |\mathbf{r} - \mathbf{r}_1|/c_0) \quad (4)$$

This can be considered as a pulse of electromagnetic field. Its propagation velocity is isotropic and remains to be c_0 in all directions in the vacuum. For time harmonic motionless sources within a bounded region, their far electric field may be expressed as

$$\mathbf{E} \approx \frac{\mathbf{E}_0}{r} \sin[\omega_0(t - r/c_0)] \quad (5)$$

where ω_0 is the angular frequency of the fields, r is the radial distance. In this case, it is a spherical wave propagating also with constant velocity c_0 in all directions.

However, in general situations with moving sources, the electromagnetic fields are the superposition of the fields from all sources at different positions and at different time instants. The composed far fields become quite complicated. Consider a point source with time varying charge $\rho_0 \cos \omega_0 t_1$, where ω_0 is the oscillating angular frequency of the charge. It moves uniformly in the vacuum with velocity $\mathbf{v}(t_1)$, as shown in Fig. 1. Its electromagnetic fields are rigorously solved from the Maxwell's equations [6, 7]. The main component of the far fields can be generally expressed in the form of

$$\mathbf{E}^{far}(\mathbf{r}, t) \approx E_0(\hat{\mathbf{n}} - \boldsymbol{\beta}) \left[\frac{1}{R(1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta})^2} \right] \sin(\omega_0 t_1) \quad (6)$$

where $\boldsymbol{\beta} = \mathbf{v}/c_0$. At time t_1 , the source moves to $\mathbf{v}t_1$. $\mathbf{R} = \mathbf{r} - \mathbf{v}t_1$ is the radius vector from the source position to the observation point. $R = |\mathbf{R}|$, and $\hat{\mathbf{n}} = \mathbf{R}/R$. The fields generated by the charge at time t_1 propagate to the observation point \mathbf{r} with a time delay of $t - t_1$.

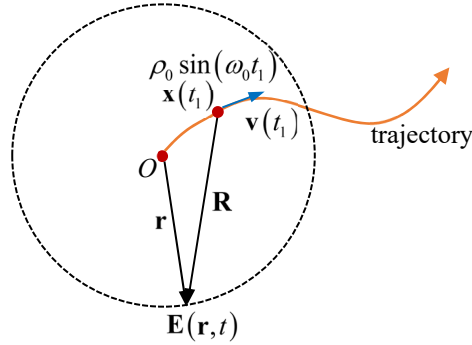


Fig.1. Trajectory of a moving point source with time-varying charge.

The behavior of the far fields is mainly characterized by the two terms: $1/R(1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta})^2$ describes the variation of the field amplitude, and $\sin(\omega_0 t_1)$ describes the propagation property of the electromagnetic wave. In particular, for the far field we have

$$t_1 \approx \frac{1}{1 - \beta^2} \left(1 + \frac{\beta \cos \theta}{\sqrt{1 - \beta^2 \sin^2 \theta}} \right) t - \frac{1}{c(1 - \beta^2)} (\beta \cos \theta + \sqrt{1 - \beta^2 \sin^2 \theta}) r \quad (7)$$

The electric far field expressed by (6) can be rewritten as

$$\mathbf{E}^{far}(\mathbf{r}, t) \approx E_0(\hat{\mathbf{n}} - \boldsymbol{\beta}) \frac{1}{R(1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta})^2} \cos[\omega(t - r/c)] \quad (8)$$

where the velocity c and the angular frequency ω of the composed far field are, respectively [7],

$$c = \frac{c_0}{\sqrt{1 - \beta^2 \sin^2 \theta}} \quad (9)$$

$$\omega = \frac{\omega_0}{1 - \beta^2} \left(1 + \frac{\beta \cos \theta}{\sqrt{1 - \beta^2 \sin^2 \theta}} \right) \quad (10)$$

The far field expressed by (8) also represents a spherical wave. However, both the frequency and the wave velocity vary with the propagation direction.

According to (9), the propagation velocity is dependent on the propagation direction. It can be seen that $c \geq c_0$ for all θ . The velocity reaches its maximum of $c_0 / \sqrt{1 - \beta^2}$ at $\theta = \pm \pi/2$. Especially, when the observer is on the path of the source, $\theta = 0, \pi$, we have to notice that the wave velocity is exactly c_0 no matter how fast the source moves.

Equation (9) clearly show that the propagation velocity of the electromagnetic waves and the moving velocity of the sources do not satisfy the vector addition rule. The statement that the two velocities can be added like vectors according to the rules of the classic physics is a server misunderstanding of the classic physics. For the sake of convenience, hereafter, we call it Misunderstanding-1.

The angular frequency ω of the far fields is also dependent on the propagation direction. This describes the Doppler effect. The normalized angular frequency is

$$s(\theta) = \frac{\omega}{\omega_0} = \frac{1}{1 - \beta^2} + \frac{\beta}{1 - \beta^2} \frac{\cos \theta}{\sqrt{1 - \beta^2 \sin^2 \theta}} \approx 1 + \beta \cos \theta + \beta^2 + \dots \quad (11)$$

The relative Doppler shift can be expressed by $s(\theta) - 1$. It is zero at the angle θ_d that satisfies

$$\cos \theta_d = -\frac{\beta}{\sqrt{1 + \beta^2}} \quad (12)$$

Particularly, when $\theta = \pm \pi/2$, the transverse Doppler effect exists

$$s\left(\pm \frac{\pi}{2}\right) = \frac{1}{1 - \beta^2} \approx 1 + \beta^2 + \dots \quad (13)$$

The Doppler effect is explicitly included in the wave solutions of the Maxwell's equations at places far away from the moving sources. Conventionally, it is stated that there is no transverse Doppler shift in the classic physics. This is another misunderstanding about the classic physics. Hereafter, we call it Misunderstanding-2.

As an example, we use the results obtained in [7], and plot the main part of the electric fields of a moving Hertzian dipole at $\beta = 0.6$ and $\beta = 0.8$ in Fig.2. The Doppler effect is clearly demonstrated.

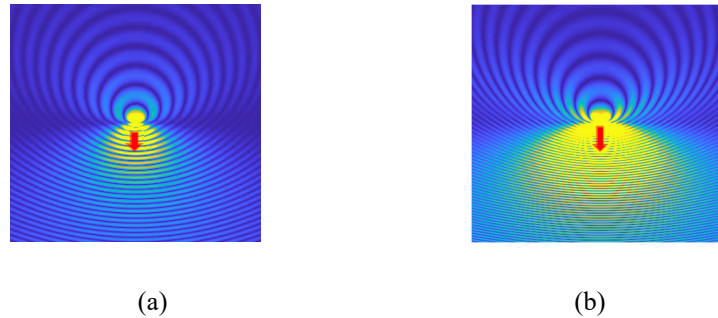


Fig.2. The field patterns of a moving Hertzian dipole. The red arrows show the moving directions. (a) $\beta = 0.6$. (b) $\beta = 0.8$.

The wave velocity of the moving Hertzian dipole for $\beta = 0.8$ is shown in Fig.3(a), with maximum value of about $1.67c_0$. The normalized frequency is shown in Fig.3(b), where the transverse Doppler shift is clearly depicted. The normalized radiation pattern is shown in Fig.3(c), which shows that the front beam is much larger than the back beam. Remind that the two beams are symmetrical for a motionless

Hertzian dipole. In the figures, the red arrows indicate the moving directions, and the blue arrow indicates the polarization direction of the Hertzian dipole.

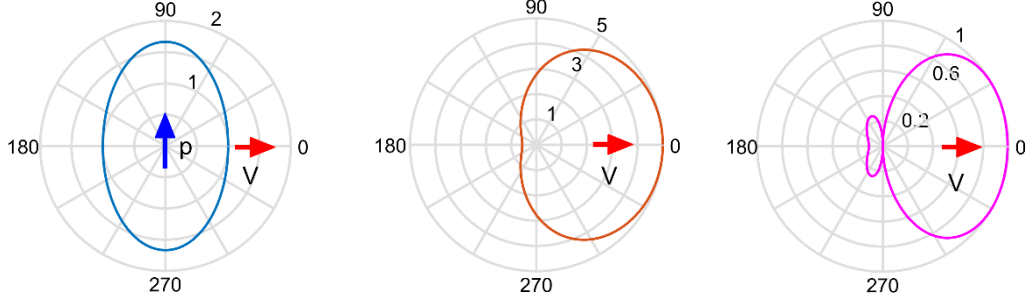


Fig. 3. A uniformly moving Hertzian dipole at $\beta = 0.8$. (a) Wave velocity. (b) Normalized frequency. (c) Normalized radiation pattern.

As a short summary, we emphasize that:

(1) $c_0 = 1/\sqrt{\mu_0 \epsilon_0}$ is a constant in the vacuum because μ_0 and ϵ_0 are constants. The propagation velocities of the electromagnetic waves are determined by the solutions to the Maxwell's equations under given sources. They are not necessarily to be c_0 for moving sources.

(2) Transverse Doppler effect is intrinsically included in the far fields of moving sources.

(3) The constant c_0 is basically valid for the electromagnetic systems that obey the Maxwell's theory.

It is not for objects that obey the Newtonian mechanics.

Maxwell's theory does not provide theoretic support to set c_0 as the velocity upper limit for all objects in the universe. Conventionally, this velocity limitation is considered to be justified by experimental results. We will show later that this is not convincing because the experiment results may have been misinterpreted.

II. Maxwell's Equations under Coordinate Transformations

The vacuum is a three-dimensional Euclidean space. We put it in the coordinate system $O(\mathbf{r}, t)$, where \mathbf{r} denotes the position of a point in the space, and t denotes the time in the coordinate system. An inertial coordinate system $O'(\mathbf{r}', t')$ moves against $O(\mathbf{r}, t)$ with velocity \mathbf{V} . Their origins coincide at $t = t' = 0$, as shown in Fig.4. The Galilean transformations describe how the position of an object changes between the two inertial reference frames,

$$\begin{cases} \mathbf{r}' = \mathbf{r} - \mathbf{V}t \\ t' = t \end{cases} \quad (14)$$

Obviously, we have $d\mathbf{r}' = d\mathbf{r}$ and $dt' = dt$, which means that the distance between two given points and the time interval between two given events are kept unchanged in all inertial reference frames.

Galilean transformations are valid for objects that are subject to the Newtonian mechanics, like stars, planes, tennis balls, and particles. The mass density $m(\mathbf{r}, t)$ of all objects should keep unchanged under the Galilean transformations,

$$m(\mathbf{r}', t') = m(\mathbf{r}, t) \quad (15)$$

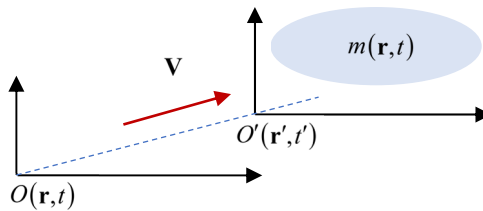


Fig.4. Two inertial coordinate systems. $O'(\mathbf{r}', t')$ moves against $O(\mathbf{r}, t)$ with velocity \mathbf{V} .

The velocity $\mathbf{v}(\mathbf{r}, t)$ and acceleration $\mathbf{a}(\mathbf{r}, t)$ are physical quantities directly related to the position \mathbf{r} and the time t . They are subject to vector addition rule and change under the Galilean transformations according to

$$\begin{cases} \mathbf{v}(\mathbf{r}', t') = \frac{d\mathbf{r}'}{dt'} = \frac{d}{dt}(\mathbf{r} - \mathbf{V}t) = \mathbf{v}(\mathbf{r}, t) - \mathbf{V} \\ \mathbf{a}(\mathbf{r}', t') = \frac{d^2\mathbf{r}'}{dt'^2} = \frac{d^2\mathbf{r}}{dt^2} = \mathbf{a}(\mathbf{r}, t) \end{cases}$$

Galileo Galilei proposed in the seventeenth century that the laws of motion are the same in all inertial reference frames. The forms of all equations in classical mechanics are kept unchanged under the Galilean transformations. This is sometimes called the Newtonian principle of relativity [4].

Electromagnetic fields are subject to the Maxwell's theory instead of the Newtonian mechanics. The Maxwell's equations in $O(\mathbf{r}, t)$ can be generally expressed by (1). It is important to emphasize that, in the vacuum, the Maxwell's equations describe the electromagnetic fields generated by the given sources $\rho(\mathbf{r}, t)$ and $\mathbf{J}(\mathbf{r}, t)$, while the sources are determined by other mechanism and not affected by the fields they have generated. The charge density is an intrinsic property of the object like the mass density. The mechanical behaviors of the charges obey the Newtonian mechanics and should keep unchanged under the Galilean transformations,

$$\rho'(\mathbf{r}', t') = \rho(\mathbf{r}, t) \quad (16)$$

However, the current density should include the effect of the motion of the coordinate system,

$$\mathbf{J}'(\mathbf{r}', t') = \rho(\mathbf{r}, t)[\mathbf{v}(\mathbf{r}, t) - \mathbf{V}] = \mathbf{J}(\mathbf{r}, t) + \rho(\mathbf{r}, t)\mathbf{V} \quad (17)$$

The current density will change in different inertial frames.

The Maxwell equations are the fundamental bases for electromagnetic systems and should not change their forms in all inertial coordinate systems. However, in the early twentieth century, researchers found that the Maxwell's equations are not covariant under the Galilean transformations. Take the first equation in (1) as an example. Under the Galilean transformations, the fields $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ in $O(\mathbf{r}, t)$ are respectively transformed to $\mathbf{E}(\mathbf{r}', t')$ and $\mathbf{B}(\mathbf{r}', t')$ in $O'(\mathbf{r}', t')$. Making use of the necessary vector identities, we get

$$\nabla' \times \mathbf{E}(\mathbf{r}', t') = (\mathbf{V} \cdot \nabla') \mathbf{B}(\mathbf{r}', t') - \frac{\partial}{\partial t'} \mathbf{B}(\mathbf{r}', t') \quad (18)$$

the form of the equation is obviously different from the first equation in (1).

In 1904, Lorentz proved that the Maxwell's equations are covariant under the Lorentz transformations (LT). Denote $\mathbf{r} = \mathbf{r}_{\parallel} + \mathbf{r}_{\perp}$, \mathbf{r}_{\parallel} and \mathbf{r}_{\perp} are respectively the component parallel to and perpendicular to \mathbf{V} . The Lorentz transformations can be written as

$$\begin{cases} \mathbf{r}'_{\parallel} = \gamma(\mathbf{r}_{\parallel} - \mathbf{V}t) \\ \mathbf{r}'_{\perp} = \mathbf{r}_{\perp} \\ t' = \gamma(t - c_0^{-2} \mathbf{r} \cdot \mathbf{V}) \end{cases} \quad (19)$$

where $\gamma = 1/\sqrt{1 - V^2/c_0^2}$. Since γ should take real value, the Lorentz transformations are only valid for $V < c_0$. Any solutions of the original Maxwell's equations for $V \geq c_0$ are prohibited by the Lorentz transformations.

With the Lorentz transformations, the electromagnetic fields are transformed with [4, 5]

$$\begin{cases} \mathbf{E}'_{\parallel}(\mathbf{r}', t') = \mathbf{E}_{\parallel}(\mathbf{r}, t) \\ \mathbf{E}'_{\perp}(\mathbf{r}', t') = \gamma(\mathbf{E}(\mathbf{r}, t) + \mathbf{V} \times \mathbf{B}(\mathbf{r}, t))_{\perp} \\ \mathbf{B}'_{\parallel}(\mathbf{r}', t') = \mathbf{B}_{\parallel}(\mathbf{r}, t) \\ \mathbf{B}'_{\perp}(\mathbf{r}', t') = \gamma(\mathbf{B}(\mathbf{r}, t) - c_0^{-2} \mathbf{V} \times \mathbf{E}(\mathbf{r}, t))_{\perp} \end{cases} \quad (20)$$

In 1905, Einstein reinterpreted the Lorentz transformations and re-derived the transformations from his two postulates of special relativity:

- (1) The laws of physics are the same in all inertial reference frames.
- (2) The velocity of light in the vacuum is the same in all inertial frames.

The first postulate is a reasonable extension from the Newtonian principle of relativity. The second postulate seems to have a strong experimental justification because there are no observation results contradicted to this postulate have been formally confirmed. However, as we have shown in the previous section, this postulate may be not convincing based on the exact solutions of the fields of moving objects. We re-checked those experiments, and strongly believe that the experimental results that supporting the light velocity constancy may have been misinterpreted. It is also possible that we may have misunderstood the Galilean transformations and the Maxwell's theory.

III. The Natural Form of the Maxwell' Equations under Galilean Transformations

In the Newtonian principle of relativity, the laws of motion are the same in all inertial reference frames. The basic idea behind this statement is that the universe is homogenous in the macroscopic scale. The observers in different places or frames perceive the universe based on the same physics laws. It seems that the most appropriate way to describe the properties of the universe is that the distance dr between two give points and the time interval dt between two give events are unchanged in all inertial frames. We consider this as the central rule in the Galilean transformations. If we choose a reference point for the position and time, then we can create a coordinate system for the universe with coordinate (\mathbf{r}, t) for an event in it. The position \mathbf{r} and time t are the zeroth-level quantities for the universe. Einstein's first postulate is a reasonable extension from the Newtonian principle of relativity. However, we think it is very important that the above-mentioned central rule should not be abandoned.

Basically, one observer is able to perceive the universe only in his own coordinate system with himself as the coordinate origin. He cannot see the universe exactly in the same way as the other observers. When the observers are close to each other, it is natural that the universe perceived by them is similar, like the people living on the Earth see a similar pattern of stars in the sky. Therefore, we may reiterate the second postulate as: all physical laws are the same for all observers. An assumption is hidden behind this statement: the observer has no impact on the physical systems he is observing.

An essential concept needs to be clarified is that the physics laws refer to the principles that describe how the physics systems evolve under certain conditions. We have to clearly distinguish the intrinsic quantities of the main bodies of the physics systems and the physical quantities used to describe the behaviors of the main bodies. We treat the mass density as the intrinsic property of the main body in a mechanical system, and the charge density as the intrinsic property in an electromagnetic system. They are the first-level physical quantities. The position, the velocity, and the momentum, etc., are all second-level physical quantities that are linearly dependent on the coordinate. They are covariant under the Galilean transformations. The acceleration, forces, potentials, fields, energies, etc., are the third-level physical quantities that are used as supplemental variables to describe the behavior of the main bodies within the frame. They are determined based on the lower-level physical quantities in the frame according to the mechanics laws or the Maxwell's theory. It is not necessary for them to be invariant under the Galilean transformations.

In particular, in electromagnetic systems, the dynamic behaviors of the charges obey the Newton's laws. The charge densities keep unchanged under the Galilean transformations and the current densities change according to (17). They are covariant under the transformations. However, the behaviors of the electromagnetic fields are subject to the Maxwell's theory instead of the Newtonian mechanics. The electromagnetic fields are determined based on the lower-level quantities in the frame according to the Maxwell's equations in that frame. They are superpositions of the fields from all sources generated at the time $t_1 \leq t$, not only those at the present time t , so are the electromagnetic potentials, electromagnetic momenta, and the electromagnetic energies. Moreover, the propagation velocity of the electromagnetic waves and the velocity of the frames simply cannot be added like vectors. Therefore, the electromagnetic fields cannot be properly addressed with the Galilean transformations. A natural idea is to find possible transformations in the whole space-time (\mathbf{r}, t) domain. It would be perfect if we can find a coordinate transformation under which all physics laws are covariant, and simultaneously, the central rule of the Galilean transformations can be observed. However, this perhaps is an extremely difficult task. The Lorentz transformations can only achieve this goal in an approximate manner: under an additional restriction that the velocity of all objects should not exceed c_0 , which obviously violates the central rule of the Galilean transformations.

The Lorentz transformations were developed when researchers found that the Maxwell's equations are not covariant under the Galilean transformations. It may have been affected by the common sense at that time that the light velocity in the vacuum is a constant, which was obviously supported by the experiment results at that time. The Lorentz transformations are basically mathematical considerations at first before Einstein gave them physical interpretations. The scientists in the physics community perceptively realized the huge merit that the Maxwell's equations are covariant under the Lorentz transformations, especially after Einstein's interpretation. It rapidly took the place of the Galilean transformations and became the base to unify the Newtonian mechanics and the Maxwell's theory. The limitation imposed by the Lorentz transformation is also accepted and gradually evolved as a consensus among almost all physics societies.

Einstein's first postulate is widely accepted and physically sound. However, as discussed in previous sections, to require that the laws of physics are the same in all inertial frames does not mean to require that the electromagnetic charges and the electromagnetic fields should be both covariant under the same coordinate transformations. It is also not necessary that the dynamic patterns of the electromagnetic systems seen by the observer in one frame should be directly mapped from the dynamic patterns in other frames through the transformations.

Based on these considerations, we propose to apply the Galilean transformations to the mechanical systems and the electromagnetic systems in the following modified way:

(1) The mass $m(\mathbf{r}, t)$ of all objects keeps unchanged under the Galilean transformations. The positions, velocities and accelerations are covariant under the transformations. Hence, the momentum also changes covariantly.

(2) The charge density is treated like the mass and keeps unchanged under the Galilean transformations. The current density changes covariantly according to (17). We can check that the current continuity law is covariant:

$$\nabla' \cdot \mathbf{J}'(\mathbf{r}', t') + \frac{\partial}{\partial t'} \rho'(\mathbf{r}', t') = \nabla \cdot \mathbf{J}(\mathbf{r}, t) + \frac{\partial}{\partial t} \rho(\mathbf{r}, t) = 0 \quad (21)$$

(3) The Maxwell's equations in $O'(\mathbf{r}', t')$ have exactly the same forms as in $O(\mathbf{r}, t)$,

$$\begin{cases} \nabla' \times \mathbf{E}'(\mathbf{r}', t') = -\frac{\partial}{\partial t'} \mathbf{B}'(\mathbf{r}', t') \\ \nabla' \times \mathbf{H}'(\mathbf{r}', t') = \frac{\partial \mathbf{D}'(\mathbf{r}', t')}{\partial t'} + \mathbf{J}'(\mathbf{r}', t') \\ \nabla' \cdot \mathbf{B}'(\mathbf{r}', t') = 0 \\ \nabla' \cdot \mathbf{D}'(\mathbf{r}', t') = \rho'(\mathbf{r}', t') \end{cases} \quad (22)$$

It is important to note that the mass, the momentum, the charge density and the current density, are transformed from that in $O(\mathbf{r}, t)$. All the electromagnetic fields do not undergo the coordinate transformations. The electromagnetic fields in $O'(\mathbf{r}', t')$ do not explicitly related to the fields in $O(\mathbf{r}, t)$, like (20) in the Lorentz transformations. The electromagnetic fields are subject to the Maxwell's equations (22) in $O'(\mathbf{r}', t')$. They are completely determined by the sources in the same inertial frame, i.e., $\rho'(\mathbf{r}', t')$ and $\mathbf{J}'(\mathbf{r}', t')$ in $O'(\mathbf{r}', t')$.

We use a point charge ρ_0 to demonstrate the relationship of the fields between two inertial frames. The charge is put at the origin of $O'(\mathbf{r}', t')$. For the observer at rest in $O'(\mathbf{r}', t')$, the magnetic field of the charge is zero, while the electric field is simply expressed by

$$\mathbf{E}'(\mathbf{r}') = \frac{\rho_0 \hat{\mathbf{r}}'}{4\pi\epsilon_0 r'^2} \quad (23)$$

For the observer at $O(\mathbf{r}, t)$, the charge moves uniformly with velocity \mathbf{V} , so the sources are

$$\begin{cases} \rho(\mathbf{r}_1, t_1) = \rho_0 \delta(\mathbf{r}_1 - \mathbf{V}t_1) \\ \mathbf{J}(\mathbf{r}_1, t_1) = \rho_0 \mathbf{V} \delta(\mathbf{r}_1 - \mathbf{V}t_1) \end{cases} \quad (24)$$

We use (\mathbf{r}_1, t_1) for the sources in $O(\mathbf{r}, t)$. The electric field and the magnetic field are derived using the Liénard-Wiechert potentials in the same frame $O(\mathbf{r}, t)$. The electric field is found to be [1, 4, 5]

$$\mathbf{E}(\mathbf{r}, t) = \frac{\rho_0}{4\pi\epsilon_0} \frac{(1 - \boldsymbol{\beta} \cdot \boldsymbol{\beta})}{R^2 (1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta})^3} (\hat{\mathbf{n}} - \boldsymbol{\beta}) \quad (25)$$

The electric fields expressed by (23) and (25) are quite different, and is not easy to explicitly map them through coordinate transformations. Note that they are the exact solutions to the Maxwell's equations in the corresponding frames. If we want to modify the solutions to include the effect of the special relativity, it means that we have to modify the Maxwell's equations themselves.

We hereafter call the modified theory as the Galilean-Newtonian-Maxwellian principle of relativity. The main points are:

(1) The distance between two give points and the time interval between two give events are unchanged in all inertial frames. There is no time dilations and no length contractions. The positions, velocities, and accelerations change with frames and observe the vector addition rule.

(2) For electromagnetic systems, the charge density $\rho(\mathbf{r}, t)$ keeps unchanged in all inertial frames. However, the current density changes with frames because of the change of the velocity of the charge.

(3) The forms of the Maxwell's equations keep unchanged in all inertial frames. The fields are determined by the sources in that frame and obey the Maxwell's equations in that frame.

(4) For mechanical systems, the mass density $m(\mathbf{r}, t)$ keeps unchanged in all inertial frames. The mass of an object is always the same value as the mass m_0 when it is at rest. The kinetic energy is expressed by $0.5mv^2$ and the momentum is mv . The forms of the Newton's laws are covariant. The mechanical behaviors of the objects obey the Newton's laws in that frame.

(5) Unlike the mass density and the charge density, the momenta and energies of the systems are conserved within one frame.

Furthermore, we add that:

(6) The causality principle should be observed in all inertial frames: the fields at time t cannot be generated by sources at time t_1 that is later than t . Explicitly, $t_1 \leq t$ is required.

In the Galilean-Newtonian-Maxwellian principle of relativity, we have applied the Einstein's first postulate in a way different from that in the theory of special relativity.

IV. Rechecking the Experimental Results

A. Light velocity constancy

Before we begin to re-check the experiments in history for measuring the light velocity constancy, we want to point out the following facts at first:

(1) The light velocity in Einstein's second postulate refers to $c(\theta)$ in (9) instead of c_0 . We can also verify this by noticing that in almost all this kind of experiments, except those using high energy particles [8, 9], interferometers are used to detect the displacement of interference fringes caused by the changes of phase differences in the light beams.

(2) In most early experiments, the results for supporting light velocity constancy were concluded based on Misunderstanding-1, which incorrectly assumes that in classic physics, the light velocity and the moving velocity of the sources or observers are added like vectors. We must re-evaluate the experiment results using the correct relationship (9) as the reference for comparison.

(3) In most early experiments, the light waves are usually converted to light beams using lens and collimators. Unfortunately, the side effect of using light beams has been largely ignored by many researchers. Theoretically, a very thin light beam can be accurately described with Gaussian beam solution under paraxial limit [2]. Near the axis of the beam, the wave front is close to that of the plane waves, and the light velocity almost equals to c_0 . In other words, converting light waves to light beams may have destroyed the original wave structures of the light. It is possible that the experiments may have measured c_0 but not $c(\theta)$. Therefore, the experiment system could not detect any displacements of the interference fringes even if its accuracy is very high.

We take several examples to show that we may have misinterpreted the experiment results [10, 11].

● Experiments of twin star

Comstock [12] and de Sitter [13] studied the orbits of binary stars to prove that the light velocity is independent on the motion of the light source. They expressed the measured light velocity as $c = c_0 \pm kv$, where v is the velocity of the twin star. They found that $k < 0.002$ and have concluded that the light velocity is independent on the speed of the twin star. However, they have drawn their conclusions based on Misunderstanding-1. Practically, if we use the correct formula (9) to predict the light velocity, we get

$k < 0.002$ when the twin star rotates with a velocity less than 1200km/s, which is much faster than the orbital speed of the Earth (29.75km/s). Even if we ignore the possible effect of using light beams in the experiments, it is still not convincing to draw the conclusion that light velocity is constant and is independent of the light sources in the vacuum. Moreover, according to (9), the largest change of the velocity is caused by the transversal component of the velocity of the twin star at $\theta = 90^\circ$. It is not caused by the radial component at $\theta = 0^\circ$ as claimed by the researchers.

- Experiments of high energy particles

Sadeh [14] measured the velocity of rays from the annihilation of positrons in flight. The center-of-mass system of the positron and electron moves with a velocity of about $0.5c_0$. Two detectors were set with angles of 20° and 135° with respect to the motion path of the positrons. This experiment shows that the velocity of the rays is independent of the velocity of the source with an experimental error of $\pm 10\%$. According to (9), we can check that the light velocity increases by 1.5% at $\theta = 20^\circ$, and by 6.9% at $\theta = 135^\circ$, both are below the experimental error. Therefore, compared to drawing conclusion that the light velocity is independent of the source, it is more reasonable to conclude that the accuracy of the experiment equipment (10%) is not high enough to detect the velocity changes ($< 6.9\%$) correctly predicted by formula (9).

Fillippas and Fox [15] measured the relative velocity of the two γ rays emitted forward and backward along the direction of the decaying π^0 meson in flight. The velocity of the neutral pions was known to be about $0.2c_0$. The experiment gives an upper limit of $k < 0.4$ with a confidence level of 90%. For comparison, the result for $0.2c_0$ predicted with (9) is about $k \approx 0.1$, which is within the upper limit. Again, the experiment supports the correctness of (9). It does not prove that the velocity of the rays is independent of the motion of the source π^0 meson, as claimed by the authors.

- Michelson-Morley type experiments

It is not necessary to check the Michelson-Morley type experiments [16, 17, 18] because when the observer is at rest with the light source, $\beta = 0$, there should be no changes of the light velocity in the vacuum, as has been correctly predicted by (9). The historical role of the Michelson-Morley type experiments may be to prove that there is no ether in the vacuum. They have achieved this goal convincingly.

- Experiments of aberrations of nebulae [11,19, 20]

When the light sources approach or leave the observer along a straight-line, then $\theta = 0$ or π , the velocity is exactly c_0 , as predicted by (9). This can explain those experiments of the aberrations of nebulae. The annual aberrations are almost constant even if the nebulae may have very large radial velocities as high as 30000km/s. This is simply because that the radial velocity has no impact on the light velocity as indicated by (9). For comparison, even if we assume that the nebula moves transversally with speed of 30000km/s, the largest light velocity is only $1.005c_0$. The aberration is about $20''.37$, not about $23''$ claimed by the authors based on Misunderstanding-1.

- The effect of the orbital velocity of the Earth

The orbital velocity of the Earth is about 29.75km/s, the largest change of the light velocity due to the orbiting of the Earth is about $\Delta c/c_0 \approx 4.9 \times 10^{-9}$, much smaller than the experimental error of most apparatus. In some experiments, to convert light waves to light beams may cause additional errors.

To summarize, if interpreted correctly, all these experimental results cannot be considered as truly supporting the light velocity constancy. On the contrary, they may be considered as good support to the proposed theory since all the results can be predicted reasonably with (9).

- Einstein's Gedanken experiment

With the proposed theory, we can explain the Einstein's Gedanken experiment [4] in a way different from special relativity. As shown in Fig.5(a), the mirror B moves uniformly with velocity \mathbf{V} . The observer A in the frame comoving with the mirror B sends a light beam to the mirror. It reflects back to A along the same path. For the observer at the comoving frame, the light travels with velocity c_0 with proper time $2t$. For the observer at the laboratory frame, the light beam travels along the path ABA' in the time interval $2t'$. According to the theory of special relativity, the time dilates in the laboratory frame, $t' = t/\sqrt{1-\beta^2}$, as shown in Fig.(b). We can check that $(c_0 t')^2 = (V t')^2 + (c_0 t)^2$, which is Einstein's

explanation. With our theory, the time interval between two given events is the same in all inertial frames, $t' = t$. However, at the laboratory frame, the light velocity along the traveling path changes to $c = c_0 / \sqrt{1 - \beta^2 \sin^2 \theta}$, where $\theta = \tan^{-1}(1/\beta)$, as shown in Fig.(c). It is straightforward to check that $(ct)^2 = (Vt)^2 + (c_0 t)^2$.

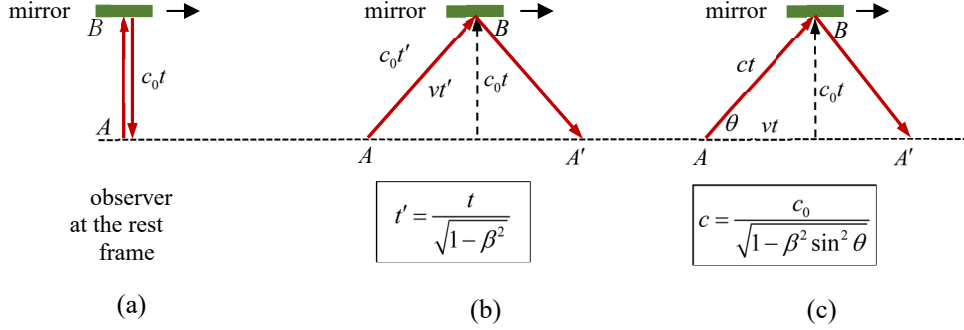


Fig. 5. Einstein's Gedanken experiment. (a) Light path in the rest frame. (b) In the laboratory frame. Explained with time dilation. (c) In the laboratory frame. Explained with anisotropic velocity.

B. Doppler effect

The formula for the Doppler effect (11) is directly derived based on the Maxwell's equations. It has clear physical meaning: the frequency of the fields may change due to the superposition of the fields from the same sources at different time instant.

In the theory of special relativity, the Doppler effect is derived according to the covariant principle that the phase of a plane wave keeps unchanged between the laboratory frame and the rest frame of the source, namely, $\omega' t' - \mathbf{k}' \cdot \mathbf{r}' = \omega t - \mathbf{k} \cdot \mathbf{r}$. It lacks the intuitive physical meaning as that derived in the classic electromagnetic theory. The Doppler effect derived based on the special relativity is [4, 11]

$$s_{SR}(\theta) = \frac{\omega}{\omega_0} = \frac{\sqrt{1 - \beta^2}}{1 - \beta \cos \theta} \approx 1 + \beta \cos \theta - \frac{1}{2} \beta^2 + \dots \quad (26)$$

The coefficient of the second-order Doppler shift is different from that in (11).

Conventionally, the transverse doppler effect is considered as a pure relativistic effect that directly related to time dilation. It has been used as a strong support to the special relativity. Now we have shown that, with the exact solution (11), this support to the special relativity is not as convincing as before.

The transverse Doppler effect was experimentally detected in 1938 by Herbert E. Ives and G.R. Stilwell using the longitudinal observation [21, 22]. Hasselkamp, Mondry, and Scharmann [23] measured the second-order Doppler shift of the H_α -line emitted by fast moving hydrogen atoms by observing the light emitted perpendicular to the direction of a linearly moving light source. These experiment results show that the coefficient of the second-order Doppler shift is about 0.5, which is close to the result predicted by the special relativity. However, in the experiments, the velocity of the source is small ($\beta < 0.031$), so the second-order Doppler shift is much smaller than the first-order one. The experimental errors are very difficult to control and evaluate, such as the error of the angle, the effect of Doppler broadening. Most importantly, the velocity of the source is evaluated based on the relativity equation $E = \gamma m_0 c^2$. It is not convincing to use the experimental results based on special relativity to support the special relativity [24]. Therefore, we think that further experiments may be required to check this issue, especially the behaviors in the situations when the velocity of the light source is comparable to c_0 .

V. Superluminal Sources

We have emphasized that the Maxwell's theory does not impose limitation to the velocity of the electromagnetic waves, let alone to the objects that do not obey the Maxwell's theory. In the proposed Galilean-Newtonian-Maxwellian principle of relativity, we lift this velocity limitation for objects, including the charges. We have shown in [7] that superluminal sources perform like electromagnetic blackholes. They can generate electromagnetic shock waves. The amplitudes, the frequencies, and the velocities all become unusually large at the edges of the shock waves. That is, the superluminal sources irradiate a large amount of high energy rays with velocity much larger than c_0 at the edges of the shock

waves. The field distributions of the shock waves generated by a Hertzian dipole moving at $1.5c_0$ and $2.0c_0$ are respectively shown in Fig.6(a) and (b). The velocity and normalized frequency for $1.5c_0$ are shown in Fig.6(c).

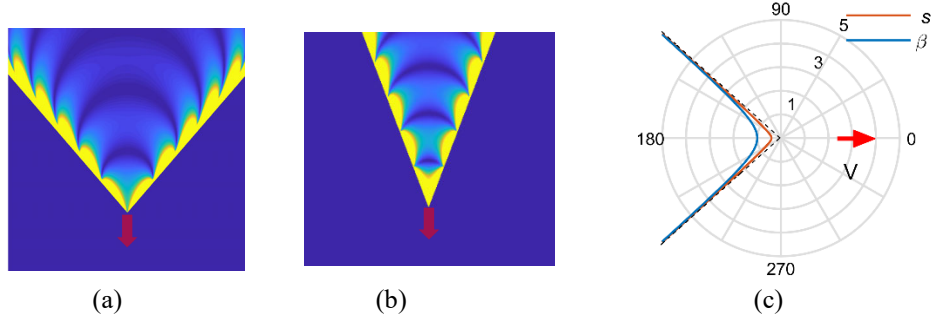


Fig. 6. Electromagnetic shock waves of a moving Hertzian dipole. (a) Field pattern at $\beta = 1.5$. (b) Field pattern at $\beta = 2.0$. (c) Wave velocity and normalized frequency at $\beta = 1.5$.

Although superluminal sources have not been officially confirmed, some cosmology events may need to be revisited [25, 26, 27]. One is the issue concerning with the lifetime of muons.

- The muons lifetime experiments

Muons were first discovered in cosmic rays. Their proper lifetime was measured to be $\tau_0 = 2.21\mu s$. Muons in cosmic rays are generally produced in the atmosphere at high altitude. However, a great part of them could travel over 10km and reach the sea level. Since $c_0\tau_0 = 660m$, the traveling path is much longer than that when the muons travel with the light velocity in the vacuum. Obviously, there are only two possibilities: muons can travel faster than light in the vacuum, or the lifetime of the flying muons increases due to the time dilation effect of special relativity. Conventionally, the first possibility is abandoned. All experimental results were explained according to the special relativity. Frisch and Smith [28] used muons produced on top of the Mt. Washington. The muons descend about 6265ft ($\approx 1900m$) toward the sea level with a speed greater than $0.99c$. About 408/563 muons can reach the sea level. Their measurements yielded an observed time dilation factor of 8.8 ± 0.8 , agree with the theoretic time dilation factor of $\gamma = 8.4 \pm 2.0$ calculated based on $E = \gamma m_0 c^2$. Therefore, the muons can travel $c_0\tau = 5800m$ and reach the sea level within their lifetime. This experiment is used to prove that time dilation is true.

In the theory we proposed, there is no time dilation. However, there is also no limit on the velocity of muons. The mass keeps unchanged in the flight. The only explanation left is that muons can travel with superluminal speed that larger than c_0 . We evaluate the velocity of the muons with Newton's mechanics, in which the kinetic energy is expressed by $E = 0.5m_0v^2$. We adopt the same parameter of $\gamma = 8.4 \pm 2.0$. By equating $E = \gamma m_0 c^2 = 0.5m_0v^2$, we can estimate that the velocity of the muons is about $4.15c_0$, and $4.15c_0\tau_0 = 2740m$. Most of the muons can also arrive the sea level if they are not absorbed by other particles in the way. Hence, the muon lifetime experiment results cannot be interpreted as only supporting the special relativity. It can also be explained with the proposed Galilean-Newtonian-Maxwellian principle of relativity.

VI. Cosmology Hypotheses

- Stellar-size blackholes [29,30]

For superluminal sources that travel with velocity greater than c_0 are not detectable if the observer stays out of the shock wave zone. These sources are electromagnetic blackholes to the observer. If we apply this hypothesis to universe, we may imagine that stellar-size blackholes may be the stars that are travelling with velocity larger than c_0 , i.e., they are superluminal stars. All stars travel with velocity smaller than c_0 are the ordinary visible stars, as shown in Fig. 7. Note that the velocity is measured in the rest frame of the observer. A blackhole to the observers on the Earth may be an ordinary star to the habitants on other planets if the velocity in their rest frames is smaller than c_0 .

- Birth of stars [29]

The superluminal stars may lose their energy and slow down. For the observers in the region outside

the shock wave zone, they are blackholes. When they slow down to cross the electromagnetic barrier, they may become suddenly detectable to the observers: the stars begin to emit light that can travel to the observers and be detected by them in the future. When the stars cross the electromagnetic barrier, they emit a large amount of high energy rays, as shown in Fig.8. For the observers, it may look like a star is just born from a blackhole, emitting a large amount of high energy rays, like γ -rays. This is a possible way of the birth of new stars in the sky: they are practically not born from blackholes but have just slowed down and crossed the electromagnetic barriers.

- High energy ray booms

When the Earth is swept by the edge of an unknown superluminal star, we may receive a strong electromagnetic boom of high energy rays. This may be one of the possible sources of the many unknown γ -ray bursts from the universe that are detected on the Earth [30].

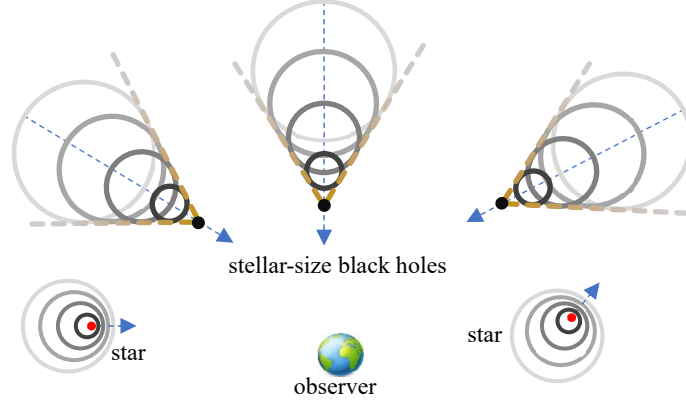


Fig. 7. Hypothesis: the stellar-size blackholes are superluminal stars.

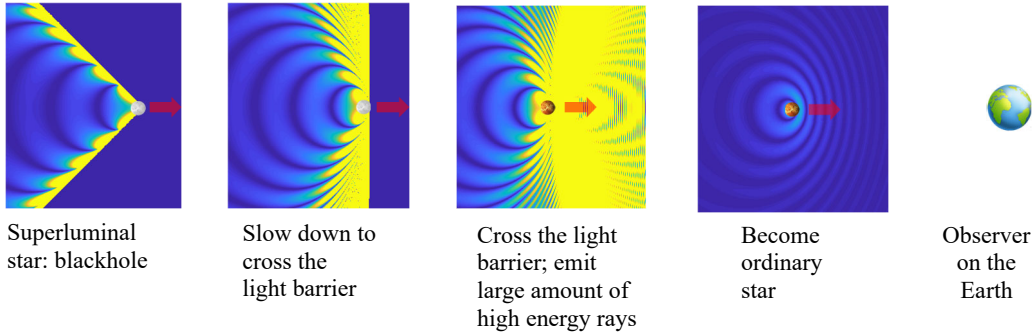


Fig. 8. Hypothesis: one possible way of stars born from the stellar-size blackholes. Superluminal stars slow down to cross the light barrier.

- Universe expansion

Since many superluminal stars and asteroids that approach the Earth with velocity larger than c_0 are invisible to us, it may cause an illusion that most of the stars are leaving us and that the universe is expanding [32], even though the universe is perhaps practically homogeneous in the macroscopic scale. The superluminal stars and asteroids may be part of the sources of the dark matters and dark energies [33] in the universe.

VII. Discussions

By correctly interpreting the exact solutions of the electromagnetic fields of moving sources, the three misunderstandings concerning about the classic physics can be clarified. It is clear that, according to the rules of the classic physics, the wave velocity and the velocity of the sources cannot be added like vectors; the light velocity in the vacuum is dependent on the propagation directions and the relative velocity between the light source and the observer, with the relationship of $c = c_0 / \sqrt{1 - \beta^2 \sin^2 \theta}$; the transverse Doppler shift is not a pure relativity effect. Therefore, it is necessary to revisit the basic rules in the classic mechanics and the classic electromagnetic theory. Naturally, it is also necessary to revisit

the Galilean transformations, the Lorentz transformations, and the Einstein's two postulates, hence, the Einstein's theory of special relativity.

We have shown that the light velocity constancy is not theoretically well-supported. By checking the experimental results for supporting the light velocity constancy, we found that almost all the experimental results were interpreted using the misunderstandings as the bases for comparison. Therefore, the light velocity constancy is also not experimentally well-supported. The Einstein's second postulate is not an unquestionable law in nature, and the Lorentz transformations is not the most favorite solution. In particular, we point out that, since the electromagnetic fields and the electromagnetic sources are subject to different physics laws, their behaviors are quite different. It is not necessary to make effort to bind them together and force them undergo the same coordinate transformations, as the Lorentz transformations do in the special relativity, at the cost of imposing a velocity limit to all objects in the universe. To state intuitively, under the Lorentz transformations, the universe is bound with the slope of the velocity limit and packaged into a four-dimensional sphere. The spacetime itself is not bent. It is just distorted under the Lorentz transformations.

Undoubtedly, it is very important to know how the main bodies are created in the universe. However, in this paper, we only discuss the mechanical systems and the electromagnetic systems in the vacuum, and start from given mass density and charge density that are not dependent on any observers.

Although superluminal sources have not been officially confirmed, it does not discourage us from performing mathematical analysis on the causal solutions of the fields of sources moving faster than the light in the vacuum. The electromagnetic shock waves are naturally introduced according to the solutions. Reasonable cosmology hypotheses can be introduced, such as superluminal stars possibly exist in the universe and may be the sources of stellar-size blackholes.

In our opinion, the theory of special relativity is an extremely powerful searchlight for scientific researches. Scientists could see through the haze hanging over modern physics at the early stages. Without its help, modern physics cannot achieve such great success. However, the theory of special relativity may have difficulties in explaining some new scientific findings in cosmology and particle physics. The light velocity limit has become a kind of forbidden line. Many findings that may violate the special relativity have to be wrapped to fit the rules of special relativity. We think it might be the time for us to give up setting the light velocity in the vacuum as the upper velocity limit for all objects in our universe. It is not necessary to use the searchlight when can see the universe in a straight way, which might be more close to the observations of our universe and time. If in the future, the spacetime is indeed found to be bent, we believe that it will be bent under physics laws that we have not yet found. It just should not be bent in the way according to the Lorentz transformations.

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